MATH 504 HOMEWORK 7

Due Friday, April 29.

Problem 1. *Suppose that* $\mathbb{P}$ *and* $\mathbb{Q}$ *are two c.c.c. posets. Show that the following are equivalent:*

1. $\mathbb{P} \times \mathbb{Q}$ *is c.c.c;*
2. $1_{\mathbb{P}} \Vdash_{\mathbb{P}} \check{\mathbb{Q}}$ *is c.c.c;*
3. $1_{\mathbb{Q}} \Vdash_{\mathbb{Q}} \check{\mathbb{P}}$ *is c.c.c;*

Problem 2. *Let* $\mathbb{P}$ *be a poset such that for every* $p \in \mathbb{P}$*, there are incompatible* $q, r \leq p$*. Suppose* G *is* $\mathbb{P}$*-generic. Show that* $G \times G$ *is not* $\mathbb{P} \times \mathbb{P}$*-generic.*

Problem 3. *Let* $\mathbb{P} \in V$ *be a poset, and let* $\dot{\mathbb{Q}}$ *be a* $\mathbb{P}$ *name for a poset, i.e.* $1_{\mathbb{P}} \Vdash_{\mathbb{P}} \dot{\mathbb{Q}}$ *is a poset. Suppose that* G *is* $\mathbb{P}$*-generic over* V*, and that* H *is* $\dot{\mathbb{Q}}_G$*-generic over* $V[G]$*. Show that* $K := G * H = \{(p, \dot{q}) \mid p \in G, \dot{q}_G \in H\}$ *is* $\mathbb{P} * \dot{\mathbb{Q}}$*-generic over* V*.*

Problem 4. *Suppose that* $\mathbb{P} * \dot{\mathbb{Q}}$ *has the* κ*-chain condition. Show that* $\mathbb{P}$ *has the* κ*-chain condition, and* $1_{\mathbb{P}} \Vdash$ *"*$\dot{\mathbb{Q}}$ *has the* κ*-chain condition".*

Remark 1. The converse is also true.

Problem 5. *Suppose that* $\mathbb{P}$ *is* κ *distributive, and* $1_{\mathbb{P}} \Vdash$ *"*$\dot{\mathbb{Q}}$ *is* κ*-distributive". Show that that* $\mathbb{P} * \dot{\mathbb{Q}}$ *is* κ*-distributive.*